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On the Pointwise Behavior of Semi-Classical Measures

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Abstract: In this paper we concern ourselves with the small $\hbar$ asymptotics of the inner products of the eigenfunctions of a Schrödinger-type operator with a coherent state. More precisely, let $\psi_j^\hbar$ and $E_j^\hbar$ denote the eigenfunctions and eigenvalues of a Schrödinger-type operator $H_\hbar$ with discrete spectrum. Let $\psi_{(x,\xi)}$ be a coherent state centered at the point (x, ξ) in phase space. We estimate as $\hbar \to 0$ the averages of the squares of the inner products $(\psi^a_{(x,\xi)}, \psi_j^\hbar)$ over an energy interval of size $\hbar$ around a fixed energy, E. This follows from asymptotic expansions of the form

$$\sum_j \varphi\left(\frac{E_j(\hbar) - E}{\hbar}\right) |(\psi^a_{(x,\xi)}, \psi_j^\hbar)|^2 \sim \sum_{k=0}^{\infty} c_k(a)\hbar^{-n+\frac{1}{2}+k}$$

for certain test functions φ and Schwartz amplitudes a of the coherent state. We compute the leading coefficient in the expansion, which depends on whether the classical trajectory through (x, ξ) is periodic or not. In the periodic case the iterates of the trajectory contribute to the leading coefficient. We also discuss the case of the Laplacian on a compact Riemannian manifold.

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