Commun. Math. Phys. 174, 191 – 214 (1995)

Communications in
Mathematical
Physics

Determinant Representation for Correlation Functions of Spin-1/2 XXX and XXZ Heisenberg Magnets

Fabian H.L. Eßler,[1,★] Holger Frahm,[2,★★] Anatoli G. Izergin,[3,★★★] Vladimir E. Korepin[4,★★★★]

[1] Physikalisches Institut der Universität Bonn, Nussallee 12, D-53115 Bonn, Germany
[2] Institut für Theoretische Physik, Universität Hannover, D-30167 Hannover, Germany
[3] Department of Mathematical Institute of Sciences, Academy of Sciences of Russia, St. Petersburg, LOMI, SU-199106, Fontanka 27, Russia
[4] Institute for Theoretical Physics, SUNY at Stony Brook, Stony Brook, NY 11794-3840, USA

Received: 16 August 1994 / in revised form: 14 December 1994

Abstract: We consider zero temperature correlation functions of the spin-$\frac{1}{2}$ XXZ Heisenberg chain in the critical regime $-1 < \Delta \leqq 1$ in a magnetic field. Starting from the algebraic Bethe Ansatz we derive representations for various correlation functions in terms of determinants of Fredholm integral operators.

1. Introduction

Despite the great advances made over the last sixty years in the study of integrable quantum models, evaluation of their correlation functions still poses a formidable problem. Quite recently there has been significant progress in this direction: the group at RIMS succeeded in deriving integral representations for some correlation functions of the Heisenberg XXZ model [2, 13, 46, 5, 16, 49–51, 56–58, 53] defined by the hamiltonian (1.1) for $\Delta > 1$ by taking advantage of the infinite quantum affine symmetry of the model on the infinite chain [10, 27]. (see e.g. [28, 12, 6, 7] for further developments). The isotropic (XXX) limit $\Delta \rightarrow 1$ was obtained in [45, 33]. These integral representations are most powerful for studying the *short distance* behaviour of correlators, whereas it is not obvious how to extract the large distance behaviour. Also it is not straightforward to extend this approach to the critical regime $-1 < \Delta < 1$ or to include an external magnetic field.

Precisely these issues can be very naturally addressed in the framework of a different approach to studying correlation functions in integrable models, which was carried out in [29, 30, 18–21, 23, 34, 35] for the example of the δ-function Bose gas [40, 41]. A detailed and complete exhibition of this work can be found in the book [32]. We call this method the *Dual Field Approach* (DFA). The DFA permits one to derive determinant representations for correlation functions of models of interacting fermions (the corresponding spectrum of the hamiltonian is not equivalent

★ E-mail: fabman@avzw02.physik.uni-bonn.de
★★ E-mail: frahm@itp.uni-hannover.de
★★★ E-mail: izergin@lomi.spb.su
★★★★ E-mail: korepin@max.physics.sunysb.edu