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The Geometry of the Quantum Correction for Topological σ-Models

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Abstract: The ring (Frobenius algebra) of local observables for topological σ-models on $\mathbb{P}^1$ with values in the grassmannian $G(s,n)$ is known to be "the same as" the quotient of the homology ring of the target space by the (inhomogeneous) ideal generated by the so-called quantum correction. While the need for a quantum correction comes from algebraic motivations in field theory, the aim of this paper is to understand its geometric meaning. The simple examples of $\mathbb{P}^1 \to \mathbb{P}^n$ models tell us that the quantum correction comes by restriction on the boundary of the moduli spaces which allows to compute intersections on moduli spaces of lower degrees. We will check this point of view for the case of $\mathbb{P}^1 \to G(s,n)$ models, yielding a proof of the algebraic result from physics in terms of the geometry of the σ-model itself.

1. Introduction

A topological field theory (TFT) is an algebraic object; it is the datum of a Frobenius algebra [D], together with its deformations. There are some TFTs directly connected with geometry (which will be called geometrical TFT or GTFT, for short). These are actually the first examples of topological field theories (see e.g [W, G, I, V]), with concrete realizations in terms of σ-models, topological Yang–Mills theory and topological gravity. The mathematical interest of these examples is that the expectation values of physical interest are actually intersection numbers in some homology rings. The ring of "topological observables" in all known GTFTs is identified in the physical literature with the quotient of the homology ring of suitable moduli spaces by the (inhomogeneous) ideal generated by the so-called "quantum correction."

We immediately have a problem: to understand for a given GTFT the geometrical origin of the quantum correction. The aim of this paper is to give an answer in the case of topological σ-models on the Riemann sphere $\mathbb{P}^1$ with values in

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