Modular Invariance and Uniqueness of Conformal Characters

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Abstract: We show that the conformal characters of various rational models of $\mathscr{W}$-algebras can be already uniquely determined if one merely knows the central charge and the conformal dimensions. As a side result we develop several tools for studying representations of $SL(2, \mathbb{Z})$ on spaces of modular functions. These methods, applied here only to certain rational conformal field theories, may be useful for the analysis of many others.

Contents

1. Introduction

In the last years two-dimensional conformal field theories played a profound role in theoretical physics as well as in mathematics. Starting with the work of A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov [1] in 1984, many new results connecting statistical mechanics and string theory with the theory of topological invariants of 3-manifolds or with number theory were found [2, 3]. In mathematical physics the classification of rational conformal field theories (RCFT) became one of the important outstanding problems.

    Since one hopes that it is possible to consider all RCFTs as rational models of $\mathscr{W}$-algebras, special vertex operator algebras generalizing in a certain sense