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Equivalence of Euclidean and Wightman Field Theories

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Abstract: A new inversion formula for the Laplace transformation of tempered distributions with supports in the closed positive semiaxis is obtained. The inverse Laplace transform of a tempered distribution is defined by means of a limit of a special distribution constructed from this distribution. The weak spectral condition on the Euclidean Green's functions implies that some of the limits needed for the inversion formula exist for any Euclidean Green's function with an even number of variables. We then prove that the initial Osterwalder–Schrader axioms [1] and the weak spectral condition are equivalent with the Wightman axioms.

1. Introduction

In 1973 K. Osterwalder and R. Schrader [1] claimed to have found necessary and sufficient conditions under which Euclidean Green's functions have analytic continuations whose boundary values define a unique set of Wightman distributions. The principal idea of the Osterwalder–Schrader paper [1] was to consider the Euclidean Green's functions to be distributions. Usually the Euclidean Green's functions were considered to be the analytic functions. Later R. Schrader [2] found a counter-example for a central lemma of the paper [1]. In 1975 K. Osterwalder and R. Schrader proposed an additional "linear growth condition" under which Euclidean Green's functions, satisfying the Osterwalder–Schrader axioms [1], define the Wightman theory. But these new extended axioms for the Euclidean Green's functions may not be equivalent with the Wightman axioms. It is possible to restore the equivalence theorem by adding the new condition [2] that the Euclidean Green's functions are Laplace transforms of the tempered distributions with supports in the positive semiaxis with respect to the time variables. The equivalence theorem then becomes trivial [2]. This new condition contradicts the main Osterwalder–Schrader idea to consider the Euclidean Green's functions to be distributions and it is not

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