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Asymptotic Expansion for the Density of States of the Magnetic Schrödinger Operator with a Random Potential

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Abstract: We study the asymptotics for the density of states of the magnetic Schrödinger operator with a random potential. By using the methods of effective Hamiltonian, complex dilation and complex translation, we obtain in the large magnetic field limit, the asymptotic expansion for the density of states measure considered as a distribution.

1. Introduction

We study the density of states of the magnetic Schrödinger operator with a random potential defined on $L^2(\mathbb{R}^2)$

$$P^{\omega}_{B,V} = (D_x + By)^2 + D_y^2 + V^{\omega}(x, y)\,,$$

where $D_x = (1/i)\partial_x$, $D_y = (1/i)\partial_y$ and $B > 0$ is a constant. Let v be a C_0^∞ function, the potential V is defined as

$$V(\bar{x}) = \sum_{i\in\mathbb{Z}^2} \alpha_i v(\bar{x} - i) = \sum_{i\in\mathbb{Z}^2} \alpha_i v_i(\bar{x})\,, \tag{1.1}$$

where $\bar{x} = (x, y)$, $\alpha = \{\alpha_i\}_{i\in\mathbb{Z}^2}$ form a random field, *i.e.* a family of random variables indexed by $\mathbb{Z}^2$ on a probability space (Ω, P). We denote by $\langle f\rangle$ the expectation value of the random variable f. One can always suppose that $\Omega = \mathbb{R}^{\mathbb{Z}^2}$. In this case,

$$\alpha_\omega(j) = \omega(j)\,, \tag{1.2}$$

and the translation operators, $T_i (i \in \mathbb{Z}^2)$ in Ω are defined by

$$T_i\omega(j) = \omega(j - i)\,. \tag{1.3}$$

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