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Fusion in the W_3 Algebra

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Abstract: We develop the notions of fusion for representations of the WA_2 algebra along the lines of Feigin and Fuchs. We present some explicit calculations for a WA_2 minimal model.

1. Introduction

The concept of fusion is central in the application of algebraic techniques in two-dimensional conformal field theory. In conformal field theory one supposes the presence of an infinite dimensional symmetry algebra, and local fields which transform under the algebra. The local fields are operator valued distributions, and it is taken as an axiom that the product of two fields may be written as a sum of fields; this is the operator product expansion. In particular there is a particular class of fields called primary fields, and in its simplest form the fusion algebra describes which irreducible representations ρ_k of the symmetry algebra can occur in the operator product of two primary fields, which we write symbolically as

$$\Phi_i \times \Phi_j \mapsto N_{ij}{}^k \rho_k \ , \qquad (1.1)$$

where $N_{ij}{}^k$ are the Verlinde fusion algebra coefficients, and are integers or infinite. For algebras with a non-zero central extension the operator product of two fields cannot simply correspond to the tensor product of two highest weight representations, as in the former case the value of the central charge is unchanged, whereas it adds under tensor product.

The simplest non-trivial algebra with which one must deal is the Virasoro algebra. Belavin, Polyakov and Zamolodchikov showed how null vectors of the Virasoro algebra affected allowed fusions [1], but Feigin and Fuchs were the first to translate their ideas into mathematical language and were able to prove the conjectured fusion rules of the Virasoro minimal models, as well as providing an algebraic definition of a minimal model in terms of a quasi-finite-representation [2]. A standard treatment would be to consider the detailed structure of the representation ρ_i in Eq. (1.1) and