Commun. Math. Phys. 170, 315 – 335 (1995)

Communications in Mathematical Physics

Classification of Bicovariant Differential Calculi on Quantum Groups

Konrad Schmüdgen, Axel Schüler

Fakultät für Mathematik und Informatik und Naturwiss., Theoretisches Zentrum, Universität Leipzig, Augustusplatz 10, D-04109 Leipzig, Germany; E-mail schmuedgen@mathematik.uni-leipzig.d400.de

Received: 16 May 1994/in revised form: 20 December 1994

Abstract: Suppose that q is not a root of unity. We classify all bicovariant differential calculi of dimension greater than one on the quantum groups $GL_q(N), O_q(N)$ and $Sp_q(N)$ for which the differentials du^i_j of the matrix entries u^i_j generate the left module of first order forms. Our first classification theorem asserts that there are precisely two one-parameter families of such calculi on $GL_q(N)$ for $N \geqq 3$. In the limit $q \to 1$ only two of these calculi give the ordinary differential calculus on $GL(N)$. Our second main theorem states that apart from finitely many q there exist precisely two differential calculi with these properties on $O_q(N)$ and $Sp_q(N)$ for $N \geqq 4$. This strengthens the corresponding result proved in our previous paper [SS2]. There are four such calculi on $O_q(3)$. We introduce two new 4-dimensional bicovariant differential calculi on $O_q(3)$.

1. Introduction

In recent years the theory of quantum groups [D,J] has attracted considerable interest among both mathematicians and theoretical physicists, cf. [Mj]. Non-commutative differential calculus on quantum groups is a fundamental tool needed for many applications. For instance, it enters essentially the formulation of gauge theory with quantum groups, see e.g. [BM or C]. A general framework for bicovariant differential calculus on quantum groups has been provided by S.L. Woronowicz [W] following general ideas of A. Connes. In contrast to the classical differential geometry on Lie groups, there is no functorial method to obtain a unique bicovariant differential calculus on a given quantum group.

In this paper we classify all bicovariant differential calculi on the quantum groups $GL_q(N), O_q(N)$ and $Sp_q(N)$ under "natural" conditions. To be precise, we assume that q is not a root of unity, the differentials $du^i_j, i,j = 1,\ldots,N$, generate the left module of the first order forms and that the dimension of the calculus is greater than one. Here $u = (u^i_j)_{i,j=1,\ldots,N}$ denotes the corresponding fundamental matrix.

The results of the present paper and its predecessor [SS2] provide a complete classification of all bicovariant differential calculi on the quantum groups $GL_q(N), SL_q(N), O_q(N)$ and $Sp_q(N)$ under the above assumptions. They show in