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Global Finite-Energy Solutions of the Maxwell–Schrödinger System

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Abstract: The existence of global finite-energy solutions is proved for the initial value problem for the Maxwell–Schrödinger system in the Coulomb, Lorentz and temporal gauges

1. Introduction

We consider the coupled Maxwell–Schrödinger system in three space dimensions for a nonrelativistic charged particle in an electromagnetic field [4]. This system occurs as a model in laser physics [1]. Although, of course, the system is not Lorentz invariant, it is rotationally invariant and gauge invariant. In this paper we prove the existence of global finite-energy solutions of the initial value problem for the Maxwell–Schrödinger system in the Coulomb gauge.

K. Nakamitsu and M. Tsutsumi in [3 and 5] proved that the initial value problem for the Maxwell–Schrödinger system in the Lorentz gauge is globally well-posed in a space of smooth functions in dimensions one and two, and locally well-posed in dimension three. Y. Tsutsumi in [6] proved, by constructing the modified wave operator, that there exist global smooth solutions in the Coulomb gauge for a certain class of scattered data as $t \to +\infty$. However, the problem in three dimensions with initial condition at a finite time has remained open. The present paper resolves this existence question, but we have not succeeded in proving uniqueness.

In Sect. 2 we write the equations and introduce the approximate system to be used to make the construction. Essentially it is to replace the imaginary i in the Schrödinger equation by $i + \varepsilon$ with a small dissipation constant ε. In Sect. 3 we construct global solutions of the approximate system in the Coulomb gauge. In Sect. 4 we pass to the limit $\varepsilon \to 0$, thereby obtaining global weak solutions. In Sect. 5 we prove the analogous result in the Lorentz gauge and in the temporal gauge.

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