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Dispersionless Toda Hierarchy and Two-Dimensional String Theory

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Abstract: The dispersionless Toda hierarchy turns out to lie in the heart of a recently proposed Landau–Ginzburg formulation of two-dimensional string theory at self-dual compactification radius. The dynamics of massless tachyons with discrete momenta is shown to be encoded into the structure of a special solution of this integrable hierarchy. This solution is obtained by solving a Riemann–Hilbert problem. Equivalence to the tachyon dynamics is proven by deriving recursion relations of tachyon correlation functions in the machinery of the dispersionless Toda hierarchy. Fundamental ingredients of the Landau–Ginzburg formulation, such as Landau–Ginzburg potentials and tachyon Landau–Ginzburg fields, are translated into the language of the Lax formalism. Furthermore, a wedge algebra is pointed out to exist behind the Riemann–Hilbert problem, and speculations on its possible role as generators of "extra" states and fields are presented.

1. Introduction

Recently a Landau–Ginzburg model of two-dimensional strings at self-dual radius (i.e., $c = 1$ topological matter coupled to two-dimensional gravity) has been proposed and studied by several groups [1, 2, 3]. This model is in a sense a natural extrapolation of the topological A_{k+1} model to $k = -3$, and seems to inherit the remarkable properties of the A_{k+1} models such as: (i) an underlying structure of Lax equation [4], (ii) a period integral representation of correlation functions [5], (iii) an algebraic structure of gravitational primaries and descendents [6], etc. Although the status of the so-called special (discrete) states [7] still remains obscure, the dynamics of massless tachyons with discrete momenta is shown to be correctly described in this new framework.

The $c = 1$ model, however, differs from the A_{k+1} (and some other $c < 1$) models in several essential aspects. This seems to be eventually due to the difference of underlying integrable hierarchies. The A_{k+1} models are special solutions of the dispersionless KP (or generalized KdV) hierarchy [8, 9, 10]. Hanany et al. [2] suggested a similar link between the $c = 1$ model and the dispersionless Toda hierarchy [11].