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Quasitriangularity of Quantum Groups at Roots of 1

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Abstract: An important property of a Hopf algebra is its quasitriangularity and it is useful for various applications. This property is investigated for quantum groups sl_2 at roots of 1. It is shown that different forms of the quantum group sl_2 at roots of 1 are either quasitriangular or have similar structure which will be called braiding. In the most interesting cases this property means that "braiding automorphism" is a combination of some Poisson transformation and an adjoint transformation with a certain element of the tensor square of the algebra.

Algebras which here will be called quantum sl_2 are the simplest examples of quantum groups which have practically all the remarkable properties of this class of Hopf algebras. One of the most important properties of quantum groups is quasitriangularity. Recall the definition from [Dr].

Definition 1. *A Hopf algebra A is called quasitriangular if there exists $R \in A \otimes A$ (or an element from the appropriate completion of $A \otimes A$) such that*

$$\Delta'(a) = R\Delta(a)R^{-1} , \tag{1}$$

$$(\Delta \otimes \mathrm{id})(R) = R_{13}R_{23} , \tag{2}$$

$$(\mathrm{id} \otimes \Delta)(R) = R_{13}R_{12} . \tag{3}$$

Here $\Delta'(a) = \sigma \circ \Delta(a)$, where $\sigma : A^{\otimes 2} \to A^{\otimes 2}$, $a \otimes b \mapsto b \otimes a$ and $R_{12}, R_{13}, R_{23} \in A^{\otimes 3}$ (or to the appropriate completion of it), $R_{12} = R \otimes 1$, $R_{23} = 1 \otimes R$, $R_{13} = (\sigma \otimes \mathrm{id})(R_{23}) = (\mathrm{id} \otimes \sigma)(R_{12})$.

A remarkable corollary of this definition is that R satisfies the Yang–Baxter equation in $A^{\otimes 3}$:

$$R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12} .$$

It is known [Dr] that quantum universal enveloping algebras $U_h\mathfrak{g}$ are quasitriangular over $\mathbb{C}[[h]]$ for any Kac–Moody algebra $\mathfrak{g}$. It is also known that the

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