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$B \wedge F$ Theory and Flat Spacetimes

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Abstract: We propose a reduced constrained Hamiltonian formalism for the exactly soluble $B \wedge F$ theory of flat connections and closed two-forms over manifolds with topology $\Sigma^3 \times (0,1)$. The reduced phase space variables are the holonomies of a flat connection for loops which form a basis of the first homotopy group $\pi_1(\Sigma^3)$, and elements of the second cohomology group of Σ^3 with value in the Lie algebra $L(G)$. When $G = SO(3,1)$, and if the two-form can be expressed as $B = e \wedge e$, for some vierbein field e, then the variables represent a flat spacetime. This is not always possible: We show that the solutions of the theory generally represent spacetimes with "global torsion." We describe the dynamical evolution of spacetimes with and without global torsion, and classify the flat spacetimes which admit a locally homogeneous foliation, following Thurston's classification of geometric structures.

1. Introduction

The $B \wedge F$ theory was first considered by Horowitz [1] as an example of an exactly soluble theory in four dimensions, analogous to the Chern–Simons formulation of (2+1)-dimensional gravity [2,3]. The set of solutions was shown to be related to equivalence classes of flat $SO(3,1)$ connections and closed two-forms. When the four-manifold has the topology $\Sigma^3 \times (0,1)$, where Σ^3 is compact and orientable, there is a natural symplectic structure which is related to the Poincaré duality between the first and second homology groups of Σ^3: Roughly speaking, the flat connections are labeled by their holonomies around loops of $Z_1(\Sigma^3)$, and the two-forms are labeled by their integrals over elements of $Z_2(\Sigma^3)$. The symplectic structure on the set of holonomies and integrated two-forms would then be derived from the Poincaré duality between $H_1(\Sigma^3) = Z_1(\Sigma^3)/B_1(\Sigma^3)$ and $H_2(\Sigma^3) = Z_2(\Sigma^3)/B_2(\Sigma^3)$.

The purpose of this article is to elucidate further the physical content of this theory. We will first postulate a reduced constrained Hamiltonian formalism [4]

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