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On Universal Vassiliev Invariants

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Abstract: Using properties of ordered exponentials and the definition of the Drinfeld associator as a monodromy operator for the Knizhnik–Zamolodchikov equations, we prove that the analytic and the combinatorial definitions of the universal Vassiliev invariants of links are equivalent.

1. Introduction

Vassiliev's knot invariants [1] contain all the invariants, such as the Jones [2], HOMFLY [3] and Kauffman [4] polynomials, which can be obtained from a deformation $U_h(\mathscr{G})$, usually called quantum group [5], of the Hopf algebra structure of enveloping algebras $U(\mathscr{G})$, where $\mathscr{G}$ is a semisimple Lie algebra.

For a compact semisimple Lie group G with Lie algebra $\mathscr{G}$, observables of the quantized Chern–Simons model give knot invariants [6] associated to $U_h(\mathscr{G})$ at special values $h = 2\pi i k^{-1}$, k a positive integer. The coefficients of the expansion in powers of h of these observables are examples of Vassiliev invariants. This is a particular case of a general theorem [7], which states that for all h the coefficients of the power series expansion of the invariants associated with semisimple Lie algebras are Vassiliev invariants.

By treating the Chern–Simons model with the conventional methods of perturbation theory, the coefficients of the powers of h of the observables can be computed [8]. Feynman diagrams and Feynman rules are the main tools of the computation. Given a knot, or more generally a link L, and the degree n (order in perturbation theory) or power of h in which one is interested, the corresponding invariant $V_n(L)$ results from the application of a Feynman rule $W_{\mathscr{G}}$ to a finite linear combination $Z_n^{CS}(L)$ of diagrams. The vector space D_n of diagrams of degree n is of finite dimension, and $Z_n^{CS}(L) \in D_n$ depends on L and on the form of the Chern–Simons action. The Feynman rule $W_{\mathscr{G}}$ depends on $\mathscr{G}$ and the representations

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