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Taming Griffiths' Singularities: Infinite Differentiability of Quenched Correlation Functions

Henrique von Dreifus[1,★], **Abel Klein**[2,★★], **J. Fernando Perez**[3,★★★]

[1] Instituto de Matemática e Estatística, Universidade de São Paulo, P.O. Box 20570, 01452-990 São Paulo, SP, Brazil
[2] Department of Mathematics, University of California, Irvine, Irvine, CA 92717-3875, USA
[3] Instituto de Física, Universidade de São Paulo, P.O. Box 20516, 01498-970 São Paulo, SP, Brazil

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Abstract: We prove infinite differentiability of the magnetization and of all quenched correlation functions for disordered spin systems at high temperature or strong magnetic field in the presence of Griffiths' singularities. We also show uniqueness of the Gibbs state and exponential decay of truncated correlation functions with probability one. Our results are obtained through new simple modified high temperature or low activity expansions whose convergence can be displayed by elementary probabilistic arguments. Our results require no assumptions on the probability distributions of the random parameters, except for the obvious one of no percolation of infinite couplings, and, in the strong field situation, for the also obvious requirement that zero magnetic fields do not percolate.

1. Introduction

In 1969 Griffiths [1] considered the statistical mechanics of a random ferromagnetic Ising model, with Hamiltonian given by

$$H = -\sum_{\langle xy \rangle} J_{xy}\sigma_x\sigma_y + h\sum_x \sigma_x \,, \tag{1.1}$$

where $x \in \mathbb{Z}^d$, $\langle xy \rangle$ denotes a pair of nearest neighbor sites in $\mathbb{Z}^d, \sigma_x = \pm 1$, and the couplings $\mathbf{J} = \{J_{xy} \geqq 0\}_{\langle xy \rangle}$ are taken as identically distributed random variables. He pointed out that for the site diluted model, i.e., $J_{xy} = J\xi_x\xi_y$, where the independent random variables ξ_x are 1 or 0 with probability p and $1-p$ respectively, the quenched magnetization, considered as a function of $z = e^{\beta h}$, displayed a non-analytical behavior at $z = 1$ for values of the inverse temperature β at which the system has neither long-range order nor spontaneous magnetization (see also [2, 3]). His arguments should apply to a large class of ferromagnetic models; in particular,

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