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Effective Masses and Conformal Mappings

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Abstract: Let G_n, $n \in \mathbf{N}$, denote the set of gaps of the Hill operator. We solve the following problems: 1) find the effective masses $M_n^{\pm}$, 2) compare the effective mass $M_n^{\pm}$ with the length of the gap G_n, and with the height of the corresponding slit on the quasimomentum plane (both with fixed number n and their sums), 3) consider the problems 1), 2) for more general cases (the Dirac operator with periodic coefficients, the Schrödinger operator with a limit periodic potential). To obtain 1)–3) we use a conformal mapping corresponding to the quasimomentum of the Hill operator or the Dirac operator.

Introduction

Consider the Hill operator $H = -d^2/dt^2 + V(t)$ in $L^2(\mathbf{R})$, where V is a 1-periodic real potential from $L^1(0,1)$. It is well known that the spectrum of H is absolutely continuous and consists of the intervals $S_1, S_2, \ldots$, and let

$$S_n = [A_{n-1}^+, A_n^-], \ldots, A_n^- \le A_n^+ < A_{n+1}^-,$$
$$n = 1, 2, \ldots, A_0^+ = 0 < A_1^-, \quad A_0^- = -\infty.$$

The intervals are separated by the gaps $G_1, G_2, \ldots$, where $G_n = (A_n^-, A_n^+)$. If a gap degenerates, i.e. $G_n = \emptyset$ then the corresponding segments S_n, S_{n+1} merge. The spectrum of the Hill operator consists of closed nonoverlapping intervals which are called spectral bands. Instead of the spectral parameter E we introduce a more convenient parameter $z, z^2 = E$, and numbers $a_n^{\pm} = \sqrt{A_n^{\pm}} \ge 0$ and gaps

$$g_n = (a_n^-, a_n^+), \quad g_{-n} = -g_n, \quad n \in \mathbf{N}, \quad g_0 = \emptyset.$$

Later on g_n will be called a gap and G_n an energy gap. Now we can define a quasimomentum function [11, 2],

$$k(z) = \arccos F(z), \quad z \in Z = \mathbf{C} \setminus \bar{g}, \quad g = \cup g_n,$$

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