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The Coloured Jones Function

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Abstract. The invariants $J_{K,k}$ of a framed knot K coloured by the irreducible $SU(2)_q$-module of dimension k are studied as a function of k by means of the universal R-matrix. It is shown that when $J_{K,k}$ is written as a power series in h with $q = e^h$, the coefficient of h^d is an odd polynomial in k of degree at most $2d + 1$. This coefficient is a Vassiliev invariant of K. In the second part of the paper it is shown that as k varies, these invariants span a d-dimensional subspace of the space of all Vassiliev invariants of degree d for framed knots. The analogous questions for unframed knots are also studied.

Introduction

A framed knot K in the 3-sphere determines an $SU(2)_q$ invariant $J_{K,k}$ for each positive integer k by using the irreducible $SU(2)_q$-module of dimension k to "colour" the knot. These invariants, sometimes called the *coloured Jones invariants* of K, are Laurent polynomials in $q^{1/4}$ with integer coefficients. Setting $q = e^h$, each coloured Jones invariant can be expanded as a rational power series

$$J_{K,k}(h) = \sum_{d=0}^{\infty} J_d(k) h^d$$

in the variable h. Together they form a single function of h and the colour k, the *coloured Jones function* of K. We shall study the dependence of this function on k.

Our main result, Theorem 1.6, is that the coefficient $J_d(k)$ of h^d in the expansion of $J_{K,k}$ is an odd polynomial in k of degree at most $2d+1$. Furthermore, if K has the zero framing then the term in k^{2d+1} vanishes, and so in this case $J_d(k)$ is of degree at most $2d-1$. An extension to the case of framed links is given in Theorem 1.7. These results have proved fruitful in our study with Kirby [7] of algebraic properties of the

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