Commun. Math. Phys. 169, 475–500 (1995)

Communications in
Mathematical
Physics

Realization of $U_q(so(N))$ within the Differential Algebra on R_q^N

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Received: 28 July 1993

Abstract. We realize the Hopf algebra $U_{q^{-1}}(so(N))$ as an algebra of differential operators on the quantum Euclidean space $\mathbf{R}_q^N$. The generators are suitable q-deformed analogs of the angular momentum components on ordinary $\mathbf{R}^N$. The algebra Fun($\mathbf{R}_q^N$) of functions on $\mathbf{R}_q^N$ splits into a direct sum of irreducible vector representations of $U_{q^{-1}}(so(N))$; the latter are explicitly constructed as highest weight representations.

1. Introduction

One of the most appealing facts explaining the present interest for quantum groups [1] is perhaps the idea that they can be used to generalize the ordinary notion of space(time) symmetry. This generalization is tightly coupled to a radical modification of the ordinary notion of space(time) itself, and can be performed through the introduction of a pair consisting of a quantum group and the associated quantum space [2, 3].

The structure of a quantum group and of the corresponding quantum space on which it coacts are intimately interrelated [2]. The differential calculus on the quantum space [4] is built so as to extend the covariant coaction of the quantum group to derivatives. Here we consider the N-dimensional quantum Euclidean space $\mathbf{R}_q^N$ and $SO_q(N)$ as the corresponding quantum group; the Minkowski space and the Lorentz algebra could also be considered, and we will deal with them elsewhere [5].

In absence of deformations, a function of the space coordinates is mapped under an infinitesimal $SO(N)$ transformation of the coordinates to a new one which can be obtained through the action of some differential operators, the angular momentum components. In other words the algebra Fun($\mathbf{R}^N$) of functions on $\mathbf{R}^N$ is the base space of a reducible representation of $so(N)$, which we can call the regular (vector) representation of $so(N)$. It is interesting to ask whether an analog of this fact occurs in the deformed case; in proper language, whether Fun($\mathbf{R}_q^N$) can be considered as a left (or right) module of the universal enveloping algebra $U_q(so(N))$, the latter being