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Generalized Bethe Ansatz Equations for Hofstadter Problem

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Abstract: The problem of diagonalization of the quantum mechanical Hamiltonian, governing dynamics of an electron on a two-dimensional triangular or square lattice in external uniform magnetic field, applied perpendicularly to the lattice plane, the flux through lattice cell, divided by the elementary quantum flux, being a rational number, is reduced to the generalized Bethe ansatz like equations on the high genus algebraic curve. Our formulae for the trigonometric case, where the genus of the curve vanishes, contain as a particular case a recent result of Wiegmann and Zabrodin.

1. Introduction

In this paper we consider the diagonalization problem of the following Hamilton operator:

$$\mathscr{H} = \mu(\alpha S + \alpha^{-1}S^{-1}) + \nu(\beta T + \beta^{-1}T^{-1}) + \rho(\gamma U + \gamma^{-1}U^{-1})\,, \tag{1.1}$$

where unitary operators S, T, and U satisfy Weyl commutation relations

$$ST = \omega TS, \quad TU = \omega UT, \quad US = \omega SU \tag{1.2}$$

with ω being a primitive N^{th} root of unity:

$$\omega = \exp(2\pi i M/N), \quad (M,N) = 1 \tag{1.3}$$

for some mutually prime integers $N > M \geqq 1$, the complex parameters α, β, γ have unit absolute values, while μ, ν and ρ are real parameters. Phases of α, β, γ are constrained to lay between 0 and $2\pi/N$:

$$0 \leqq \arg(\alpha), \quad \arg(\beta), \quad \arg(\gamma) \leqq 2\pi/N\,. \tag{1.4}$$

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