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Group Actions on C*-Algebras, 3-Cocycles and Quantum Field Theory

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Abstract: We study group extensions $\Delta \to \Gamma \to \Omega$, where Γ acts on a C*-algebra $\mathscr{A}$. Given a twisted covariant representation π, V of the pair $\mathscr{A}$, Δ we construct 3-cocycles on Ω with values in the centre of the group generated by $V(\Delta)$. These 3-cocycles are obstructions to the existence of an extension of Ω by $V(\Delta)$ which acts on $\mathscr{A}$ compatibly with Γ. The main theorems of the paper introduce a subsidiary invariant Λ which classifies actions of Γ on $V(\Delta)$ and in terms of which a necessary and sufficient condition for the the cohomology class of the 3-cocycle to be non-trivial may be formulated. Examples are provided which show how non-trivial 3-cocycles may be realised. The framework we choose to exhibit these essentially mathematical results is influenced by anomalous gauge field theories. We show how to interpret our results in that setting in two ways, one motivated by an algebraic approach to constrained dynamics and the other by the descent equation approach to constructing cocycles on gauge groups. In order to make comparisons with the usual approach to cohomology in gauge theory we conclude with a Lie algebra version of the invariant Λ and the 3-cocycle.

1. Introduction

Group three cocycles arise from the descent equation approach to the study of the cohomology of gauge groups. This early work was motivated by the need to understand anomalies in gauge theories [2, 4, 10, 26]. From the viewpoint of Dirac's constrained dynamical systems these are models with second class constraints. There have been many attempts to explain and interpret these 3-cocycles [2, 3, 4, 10, 25, 26, 30]. For example Dirac's quantisation condition for the charge of a magnetic monopole [3, 10, 30] has been interpreted as the vanishing of a 3-cocycle whilst non-vanishing 3-cocycles have been interpreted as nonassociative algebra multiplication [4, 10, 26]. Neither of these approaches has yielded to mathematical analysis. In [4] the conventional mathematical interpretation of 3-cocycles as obstructions was described, but the question remained: obstructions to what? In [1] one of us pointed out an interpretation of a 3-cocycle on a group of