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The Broken Supersymmetry Phase of a Self-Avoiding Random Walk

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Abstract: We consider a weakly self-avoiding random walk on a hierarchical lattice in $d = 4$ dimensions. We show that for choices of the killing rate a less than the critical value a_c the dominant walks fill space, which corresponds to a spontaneously broken supersymmetry phase. We identify the asymptotic density to which walks fill space, $\rho(a)$, to be a supersymmetric order parameter for this transition. We prove that $\rho(a) \sim (a_c - a)\,(-\log(a_c - a))^{1/2}$ as $a \nearrow a_c$, which is mean-field behavior with logarithmic corrections, as expected for a system in its upper critical dimension.

1. Introduction and Results

The self-avoiding walk (SAW) has long been studied in the physics literature due to its significance as a model for physical polymers [dG2, dCJ]. Recently it has received attention from a rigorous perspective as well [MS, BI, IM]. Most of the rigorous work has been directed towards establishing the properties of either fixed-length walks in the presence of a strictly repulsive interaction or the Green's function of such a process at or above the critical point. In this paper, however, we study a SAW in the so-called dense phase, where the dominant paths fill space to some nonzero mean density. We work in $d = 4$ dimensions, which is the borderline between simple mean-field behavior ($d > 4$) and complex behavior ($d < 4$). A consequence of this is that the critical behavior is modified slightly from mean-field, but is still tractable. For a weakly self-avoiding walk on a hierarchical lattice we rigorously calculate the critical behavior of the density, finding the leading power-law behavior to be mean-field, but with logarithmic corrections.

The model we study is essentially the same as the one introduced in [BEI, BI], so we will only briefly describe it here. By a hierarchical lattice $\mathscr{G}$ we mean the direct sum of infinitely many copies of $\mathbb{Z}_{L^4}$, with L some positive integer. A point

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