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q-Lorentz Group and Braided Coaddition on q-Minkowski Space

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Abstract: We present a new version of q-Minkowski space, which has both a coaddition law and an $SL_q(2,\mathbb{C})$-spinor decomposition. The additive structure forms a braided group rather than a quantum one. In the process, we obtain a q-Lorentz group which coacts covariantly on this q-Minkowski space.

1. Introduction

In recent years, there has been some speculation whether it could be possible to regularise singularities in quantum field theories by making spacetime slightly non-commutative. As well as the programme of A. Connes [3] based on the theory of operator algebras, there is also a more naive approach based on the idea of q-deformation. In this approach, which is the one we shall follow, non-commutativity is controlled by a parameter q such that one recovers the commutative case for $q = 1$. This programme is motivated by examples of "Feynman-type" integrals over two-dimensional q-deformed planes which are of the form $\int(\ldots) = \frac{1}{q^2-1}(finite)$, i.e. are divergent only in the commutative case [7]. Moreover, one hopes in such a q-regularisation scheme to preserve all symmetries as q-symmetries, using the standard techniques for q-deforming Lie algebras, etc. One would then set $q = 1$ after intelligent renormalisation, although, to take account of Planck scale corrections to the geometry, one might even keep $q \neq 1$.

As an important element of such a q-regularisation scheme, many q-Lorentz groups and q-Minkowski spaces have been recently proposed [17, 2, 16, 15]. One of the points of view in these works, which will be our point of view also, is that q-Minkowski space should have a q-spinor decomposition. Mathematically, q-Minkowski space should be a q-deformed version of 2×2 Hermitean matrices and the q-Lorentz group should act on it by conjugation by two q-deformed $SL(2,\mathbb{C})$ transformations. The rôle of such a q-deformed $SL(2,\mathbb{C})$ can be provided by the quantum double [17], but q-Minkowski space and the q-Lorentz group itself are less well understood so far.

Naively, one might try to construct q-Minkowski space as quantum 2×2 matrices, but this algebra is not covariant under the coaction of the q-deformed $SL(2,\mathbb{C})$