Commun. Math. Phys. 168, 227–247 (1995)

Communications in Mathematical Physics
© Springer-Verlag 1995

U (1) Gauge Theory on a Torus

Yu. M. Zinoviev[⋆,⋆⋆]
Steklov Mathematical Institute, Vavilov St. 42, Moscow 117966, GSP-1, Russia

Received: 8 February 1993/in revised form: 26 April 1994

Abstract: $U(1)$ gauge theory with the Villain action on a cubic lattice approximation of three- and four-dimensional torus is considered. As the lattice spacing approaches zero, provided the coupling constant correspondingly approaches zero, the naturally chosen correlation functions converge to the correlation functions of the **R**-gauge electrodynamics on three- and four-dimensional torus. When the torus radius tends to infinity these correlation functions converge to the correlation functions of the **R**-gauge Euclidean electrodynamics.

1. Introduction

The compact lattice gauge field theory models introduced by K. Wilson [1] preserve the differential geometric structures of the continuum theory. This paper is concerned with the case where the gauge group is $U(1) = \mathbf{R}/2\pi\mathbf{Z}$. Let $h(\theta)$ be a real twice continuously differentiable even periodic function with period 2π. Any such function will be called an energy function. The main examples of interest are the Wilson [1] energy function $h(\theta) = 1 - \cos\theta$ and the Villain [2] energy function

$$\exp\,[-\beta h_\beta(\theta)] = c_\beta \sum_{n=-\infty}^{\infty} \exp\,[-\beta(\theta - 2\pi n)^2/2]\,, \tag{1.1}$$

where $\beta > 0$ and c_β is a constant chosen so that the right-hand side is one for $\theta = 0$.

Let e_i, $i = 1,\ldots,d$ be the standard unit vectors in $\mathbf{R}^d$, and p be a non-negative integer less than d. The p-cells based at $\mathbf{m} \in \mathbf{Z}^d$ are the formal symbols: $(\mathbf{m}; e_{i_1},\ldots,e_{i_p})$, where the unit vectors differ from each other.

Let G be one of three abelian groups: $\mathbf{Z}$, $\mathbf{R}$ and $U(1) = \mathbf{R}/2\pi\mathbf{Z}$. A p-cochain with the coefficients in G is a G-valued function on p-cells $f(\mathbf{m}; e_{i_1},\ldots,e_{i_p}) \equiv f_{i_1\ldots i_p}(\mathbf{m})$ which is antisymmetric under the permutations of the indices $i_1,\ldots,i_p$.

⋆ Supported by the Russian Foundation of Fundamental Researches under Grant 93-011-147
⋆⋆ e-mail: zinoviev@mian.su