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Classical limit of the Quantized Hyperbolic Toral Automorphisms

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Abstract: The canonical quantization of any hyperbolic symplectomorphism A of the 2-torus yields a periodic unitary operator on a N-dimensional Hilbert space, $N = \frac{1}{h}$. We prove that this quantum system becomes ergodic and mixing at the classical limit ($N \to \infty$, N prime) which can be interchanged with the time-average limit. The recovery of the stochastic behaviour out of a periodic one is based on the same mechanism under which the uniform distribution of the classical periodic orbits reproduces the Lebesgue measure: the Wigner functions of the eigenstates, supported on the classical periodic orbits, are indeed proved to become uniformly spread in phase space.

Contents

0. Introduction

The quantization of any linear hyperbolic symplectomorphism A of the 2-torus $\mathbb{T}^2$ yields a unitary operator V_A acting on a Hilbert space of finite dimension $N = h^{-1}$, in agreement with the well known physical intuition that a compact phase space