

Classical limit of the Quantized Hyperbolic Toral Automorphisms

Mirko Degli Esposti, Sandro Graffi, Stefano Isola

Dipartimento di Matematica, Università di Bologna, I-40127 Bologna, Italy

Received: 15 September 1993/in revised form: 10 February 1994

Abstract: The canonical quantization of any hyperbolic symplectomorphism A of the 2-torus yields a periodic unitary operator on a N -dimensional Hilbert space, $N = \frac{1}{h}$. We prove that this quantum system becomes ergodic and mixing at the classical limit ($N \rightarrow \infty$, N prime) which can be interchanged with the time-average limit. The recovery of the stochastic behaviour out of a periodic one is based on the same mechanism under which the uniform distribution of the classical periodic orbits reproduces the Lebesgue measure: the Wigner functions of the eigenstates, supported on the classical periodic orbits, are indeed proved to become uniformly spread in phase space.

Contents

1. Von Neumann definition of the quantum ergodicity and mixing properties. Statement of the main results.	473
2. Koopman operator on invariant lattices and periodic orbits. Limits of atomic invariant measures supported on periodic orbits via Kloosterman sums.	477
3. Quantization of toral automorphisms. Discrete Wigner functions. Support on classical periodic orbits, relation with the Koopman operator and explicit construction of the quantum eigenvectors.	482
4. Classical limit of the matrix elements of the observables via Weil–Deligne exponential sums. Weak-* convergence of the Wigner functions. Proof of the main results.	495
Appendix A. Some basic results out of number theory.	503

0. Introduction

The quantization of any linear hyperbolic symplectomorphism A of the 2-torus \mathbb{T}^2 yields a unitary operator V_A acting on a Hilbert space of finite dimension $N = h^{-1}$, in agreement with the well known physical intuition that a compact phase space