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Nonintegrability of Some Hamiltonian systems, Scattering and Analytic Continuation

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Abstract: We consider canonical two degrees of freedom analytic Hamiltonian systems with Hamiltonian function $H=\frac{1}{2}[p_1^2+p_2^2]+U(q_1,q_2)$, where $U(q_1,q_2)=\frac{1}{2}[-v^2q_1^2+\omega^2q_2^2]+\mathcal{O}(q_1^2+q_2^2)^{3/2})$ and $\partial_{q_2}U(q_1,0)=0$. Under some additional, not so restrictive hypothesis, we present explicit conditions for the existence of transversal homoclinic orbits to some periodic orbits of these systems. We use a theorem of Lerman (1991) and an analogy between one of its conditions with the usual one dimensional quantum scattering problem. The study of the scattering equation leads us to an analytic continuation problem for the solutions of a linear second order differential equation. We apply our results to some classical problems.

1. Introduction

Hamiltonian systems are usually classified as integrable or nonintegrable. *In this article we restrict our attention to two degrees of freedom real analytic Hamiltonian systems and say that a system is integrable if it has an analytic first integral independent of its Hamiltonian function.* If the system is integrable then its dynamics is essentially almost periodic and we can say that it is "well-behaved." Integrability is a very strong property and integrable systems are the exception. So, why do we care about integrable systems? Besides the fact that integrable systems appear in some physical models, a possible answer is because this is the unique situation where we have a more or less complete description of the global dynamics. In the nonintegrable case the dynamics is much more complicated, and a usual approach is to consider the nonintegrable system as some perturbation of an integrable one. Due to the "practical" importance of the integrable systems it is crucial to decide if a given system is or is not integrable.

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