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Staggered Polarization of Vertex Models with $U_q(\widehat{sl}(n))$-Symmetry

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Abstract: In this paper we give an explicit formula for level 1 vertex operators related to $U_q(\widehat{sl}(n))$ as operators on the Fock spaces. We derive also their commutation relations. As an application we calculate with the vector representation of $U_q(\widehat{sl}(n))$, thereby extending the recent work on the staggered polarization of the XXZ-model.

1. Introduction

The Hamiltonian of the XXZ-model has $U_q(\widehat{sl}(2))$-symmetry in the thermodynamic limit. Recently, on the basis of this fact, the XXZ-model was formulated in the framework of representation theory of $U_q(\widehat{sl}(2))$. Let us explain the scheme described in [1] briefly.

First we recall XXZ-model as it appears in physics. The space of states of the XXZ-model is the infinite tensor product $\cdots \otimes V \otimes V \otimes V \otimes \cdots$, where $V = \mathbf{C}v_+ \otimes \mathbf{C}v_-$ is the two-dimensional vector space. The XXZ-Hamiltonian is the following operator formally acting on the above space:

$$H_{XXZ} = -\frac{1}{2}\sum_{k\in\mathbf{Z}} (\sigma_k^x \sigma_{k+1}^x + \sigma_k^y \sigma_{k+1}^y + \frac{q+q^{-1}}{2}\sigma_k^z \sigma_{k+1}^z)\,,$$

where σ^x, σ^y, σ^z are the Pauli matrices on V, σ_k^α acting on the k^{th} component of $\cdots \otimes V \otimes V \otimes V \otimes \cdots$. Let $U_q'(\widehat{sl}(2))$ denote the subalgebra of $U_q(\widehat{sl}(2))$ with the grading operator d being dropped. It acts on V as follows:

$$e_1 \cdot v_- = v_+\,, \quad f_1 \cdot v_+ = v_-\,, \quad t_1 \cdot v_\pm = q^{\pm 1} v_\pm\,,$$

$$e_0 \cdot v_+ = v_-\,, \quad f_0 \cdot v_- = v_+\,, \quad t_0 \cdot v_\pm = q^{\mp} v_\pm\,.$$

$U_q'(\widehat{sl}(2))$ acts on $\otimes V \otimes V \otimes V \otimes \cdots$ via the iterated coproduct $\Delta^{(\infty)}$.

$$\Delta^{(\infty)}(t_i) = \cdots \otimes t_i \otimes t_i \otimes t_i \otimes \cdots\,,$$

$$\Delta^{(\infty)}(e_i) = \sum \cdots \otimes t_i \otimes t_i \otimes e_i \otimes 1 \otimes 1 \otimes \cdots\,,$$

$$\Delta^{(\infty)}(f_i) = \sum \cdots \otimes 1 \otimes 1 \otimes f_i \otimes t_i^{-1} \otimes t_i^{-1} \otimes \cdots\,.$$