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Effective Action in Spherical Domains

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Abstract: The effective action on an orbifolded sphere is computed for minimally coupled scalar fields. The results are presented in terms of derivatives of Barnes ζ-functions and it is shown how these may be evaluated. Numerical values are shown. An analytical, heat-kernel derivation of the Cesàro-Fedorov formula for the number of symmetry planes of a regular solid is also presented.

1. Introduction

In an earlier work [1] we have shown that the ζ-function, $\zeta_\Gamma(s)$, on orbifold-factored spheres, S^d/Γ, for a conformally coupled scalar field, is given by a Barnes ζ-function, [2], $\zeta_d(s, a|\,\mathbf{d})$, where the d_i are the *degrees* associated with the tiling group Γ. The *free-field Casimir energy on the space-time* $\mathbb{R} \times S^d/\Gamma$ *was given as the value of the* ζ-function at a negative integer which evaluated to a generalised Bernoulli function. In the present work we wish to consider the effective action on orbifolds S^d/Γ which this time are to be looked upon as Euclidean space-times. In particular we will discuss $d = 2$ and $d = 3$, concentrating on the former.

The simplifying assumption in our previous work was that of conformal coupling on $\mathbb{R} \times S^d/\Gamma$. This made the relevant eigenvalues perfect squares and allowed us to use known generating functions to incorporate the degeneracies. From the point of view of field theories on the space-times S^d/Γ, retaining this assumption would be rather artificial. A more appropriate choice would be minimal coupling, or possibly conformal coupling, on S^d/Γ. (These coincide for $d = 2$.)

The quantities in which we are interested are $\zeta'_\Gamma(0)$ and $\zeta_\Gamma(0)$. The latter determines the divergence in the effective action and the former is, up to a factor and a finite addition, the renormalised effective action (i.e. half the logarithm of the functional determinant).