Commun. Math. Phys. 162, 399–431 (1994)

Communications in Mathematical Physics

On the Completeness of the Set of Classical $\mathcal{W}$-Algebras Obtained from DS Reductions

L. Fehér [1, ⋆], **L. O'Raifeartaigh** [2], **P. Ruelle** [2], **I. Tsutsui** [2]

[1] Physikalisches Institut der Universität Bonn, Nußallee 12, D-53115 Bonn, Germany
[2] Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland

Received: 30 April 1993/in revised form: 9 September 1993

Abstract: We clarify the notion of the DS – generalized Drinfeld-Sokolov – reduction approach to classical $\mathcal{W}$-algebras. We first strengthen an earlier theorem which showed that an $sl(2)$ embedding $\mathcal{S} \subset \mathcal{G}$ can be associated to every DS reduction. We then use the fact that a $\mathcal{W}$-algebra must have a quasi-primary basis to derive severe restrictions on the possible reductions corresponding to a given $sl(2)$ embedding. In the known DS reductions found to date, for which the $\mathcal{W}$-algebras are denoted by $\mathcal{W}_{\mathcal{S}}^{\mathcal{G}}$-algebras and are called canonical, the quasi-primary basis corresponds to the highest weights of the $sl(2)$. Here we find some examples of noncanonical DS reductions leading to $\mathcal{W}$-algebras which are direct products of $\mathcal{W}_{\mathcal{S}}^{\mathcal{G}}$-algebras and "free field" algebras with conformal weights $\Delta \in \{0, \frac{1}{2}, 1\}$. We also show that if the conformal weights of the generators of a $\mathcal{W}$-algebra obtained from DS reduction are nonnegative $\Delta \geq 0$ (which is the case for all DS reductions known to date), then the $\Delta \geq \frac{3}{2}$ subsectors of the weights are necessarily the same as in the corresponding $\mathcal{W}_{\mathcal{S}}^{\mathcal{G}}$-algebra. These results are consistent with an earlier result by Bowcock and Watts on the spectra of $\mathcal{W}$-algebras derived by different means. We are led to the conjecture that, up to free fields, the set of $\mathcal{W}$-algebras with nonnegative spectra $\Delta \geq 0$ that may be obtained from DS reduction is exhausted by the canonical ones.

1. Introduction

The study of nonlinear extensions of the Virasoro algebra by conformal primary fields was initiated by A.B. Zamolodchikov in the pioneering paper [1]. Such algebras, known as $\mathcal{W}$-algebras, play an important rôle in two dimensional conformal field theories, gravity models and integrable systems. (For detailed reviews, see e.g. [2, 3].) At least three distinct methods are used in the literature for constructing $\mathcal{W}$-algebras. These can be labelled as direct constructions [1, 4, 5], the methods of extracting $\mathcal{W}$-algebras from conformal field theories (the most important of which is the coset

⋆ On leave from Bolyai Institute of Szeged University, H-6720 Szeged, Hungary