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Analysis of the Static Spherically Symmetric $SU(n)$-Einstein-Yang-Mills Equations

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Abstract: The singular boundary value problem that arises for the static spherically symmetric $SU(n)$-Einstein-Yang-Mills equations in the so-called magnetic case is analyzed. Among the possible actions of $SU(2)$ on a $SU(n)$-principal bundles over space-time there is one which appears to be the most natural. If one assumes that no electrostatic type component is present in the Yang-Mills fields and the gauge is suitably fixed a set of $n-1$ second order and two first order differential equations is obtained for $n-1$ gauge potentials and two metric components as functions of the radial distance. This system generalizes the one for the case $n=2$ that leads to the discrete series of the Bartnick-Mckinnon and the corresponding black hole solutions. It is highly nonlinear and singular at $r=\infty$ and at $r=0$ or at the black hole horizon but it is known to admit at least one series of discrete solutions which are scaled versions of the $n=2$ case. In this paper local existence and uniqueness of solutions near these singular points is established which turns out to be a nontrivial problem for general n. Moreover, a number of new numerical soliton (i.e. globally regular) numerical solutions of the $SU(3)$-EYM equations are found that are not scaled $n=2$ solutions.

1. Introduction

The coupling of Einstein's general relativity with Yang-Mills gauge theories leads to complicated nonlinear systems of equations even in the static spherically symmetric case. If the gauge group is $SU(2)$ and the "Coulomb" part of the gauge potential is set to zero and asymptotical flatness is imposed the resulting singular boundary value problem admits a sequence of regular solutions parametrized by the number of zeros of a convenient gauge potential component. These solutions were numerically discovered by Bartnik and Mckinnon [3] and their existence was proved analytically by Smoller et al. [18–20] for some range of the initial conditions for a suitable gauge potential at the center or at the black hole horizon. Such discrete sequences of solutions have since also been found numerically for a number of other field