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Purification and Uniqueness of Quantum Gibbs States

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Abstract: We give a new condition for uniqueness of Gibbs states of quantum spin models on lattices.

1. Introduction

In this paper, we propose a new method for proving uniqueness of Gibbs states of quantum spin models on lattices. Our method is based on the ergodicity of Markov semigroups (semigroups of completely positive maps on C^*-algebras). This is a standard method in classical spin models, but has never been developed in quantum cases. Our motivation was to find a quantum analogue of Dobrushin's uniqueness condition for Gibbs measure ([4 and 5]), though the condition we obtained in this paper is still a perturbation theory near the infinite temperature.

In our opinion, we encounter two types of difficulties originating from the non-commutativity when considering the uniqueness theorem for quantum models.

(i) The problem of entire analyticity: Let $\alpha_t(Q)$ be the time evolution of a local observable $\mathbf{Q}$, formally written as $\alpha_t(Q)=e^{itH}Qe^{-itH}$, where H is the infinite volume Hamiltonian. The notion of quantum DLR equation was first introduced in [1]. The DLR equation for a quantum state φ is

$$\varphi(Q)=\mathrm{tr}_I\otimes\psi(\Lambda_I^* Q\Lambda_I)\,, \tag{1}$$

where tr_I is the normalized trace on the volume I, ψ is a (possibly non-normalized) positive linear functional of the observables supported outside I, and Λ_I is an operator given by the help of the relative modular operator of the modular theory of von Neumann algebras. Λ_I has a formal expression,

$$\Lambda_I=\exp(-1/2(H-H_I))\exp(1/2H)\,, \tag{2}$$

where H_I is the finite Hamiltonian on the volume I with boundary terms.

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