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Chern–Simons Theory, Coloured-Oriented Braids and Link Invariants

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Abstract: A method to obtain explicit and complete topological solution of SU(2) Chern–Simons theory on S^3 is developed. To this effect the necessary aspects of the theory of coloured-oriented braids and duality properties of conformal blocks for the correlators of $SU(2)_k$ Wess–Zumino conformal field theory are presented. A large class of representations of the generators of the groupoid of coloured-oriented braids are obtained. These provide a whole lot of new link invariants of which Jones polynomials are the simplest examples. These new invariants are explicitly calculated as illustrations for knots up to eight crossings and two-component multicoloured links up to seven crossings.

1. Introduction

Topological quantum field theories provide a bridge between quantum physics on one hand and geometry and topology of low dimensional manifolds on the other [1]. The functional integral formulation of such quantum field theories provides a framework to study this relationship. In particular, a class of topological field theories which are related to knot theory have attracted a good deal of attention in recent times. This started with the seminal work of Witten who not only put the Jones polynomials [2] in a field theoretic setting, but also presented a general field theoretic framework in which knot theory could be studied in an arbitrary three-manifold [3].

In $SU(2)$ Chern–Simons gauge theory, the expectation value of Wilson link operators with doublet representation placed on all the component knots yields Jones polynomials. Two variable generalization of these polynomials, the so-called HOMFLY polynomials [4], are obtained as the expectation value of Wilson link operators with N dimensional representation on all the component knots in an $SU(N)$ Chern–Simons theory. In fact Witten [3] has shown that the expectation values of such link operators obey the same Alexander–Conway skein relation as those by Jones and HOMFLY polynomials respectively. These relations can be recursively solved to obtain these polynomials for an arbitrary link. Placing