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Ferretti–Rajeev Term and Homotopy Theory

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Abstract: We reduce Ferretti–Rajeev models to the usual sigma models with Chern–Simons terms (θ-terms), and show that whether θ is quantized or not corresponds to the fact $\pi_4(G_{j,n}) \cong \pi_3(U(j)) = \mathbb{Z}$ or 0 of the topology in the process of our reduction. We also reconsider the topological invariance of the Chern classes in the language of the field theory.

Introduction

Two dimensional chiral models have widely been studied from both the classical and quantum point of view. See, for example, [16] and its references. But new developments were brought by Witten [15]. Namely he added the so-called Wess–Zumino–Witten term to the kinetic one of chiral models. Then the conformal invariance of the theory was recovered and the boson-fermion correspondence in the quantum level was shown. Unfortunately it is not easy to extend these models in higher dimensions. See, for example, [10].

Next two dimensional nonlinear Grassmann σ models have also been studied from both classical and quantum levels and many interesting results have been obtained, see [16]. These models are known as a good toy model for QCD.

Recently Ferretti–Rajeev [3] tried to give a Wess–Zumino–Witten like approach to these models. Namely they proposed new models adding the 2nd Chern class to the kinetic term (of these models). Unlike the Wess–Zumino–Witten models, these ones, of course, do not have the conformal invariance. But these ones are renormalizable in the $1/N$-expansion, see [3].

They have also rewritten their Grassmann models with chiral fields, and gave current algebra forms in 3-dimensions. In passing to the quantum level, these algebras must be made the abelian extension (not central extension in the case of WZW models), so that it is not easy to treat. See [11], [7].

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