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On the Spherical Symmetry of Static Stellar Models

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Abstract: This paper completes the proof of the necessity of spherical symmetry in the static general-relativistic stellar models that have equations of state satisfying certain inequalities. The technical assumption – that there exists a "reference spherical stellar model" – that was essential in the previous discussions of this problem is removed. This paper also extends beyond previous discussions the class of equations of state included in the proof. The analysis of the equations for spherical stellar models, used here to demonstrate the existence of a "reference spherical model," may also be of independent interest.

1. Introduction

It seems almost self evident that spherical symmetry is a necessary feature of any equilibrium stellar model which is nonrotating, self gravitating, physically isolated, and composed entirely of fluid. The proof of this "obvious" fact for Newtonian stellar models – although far from trivial – was given many years ago (Lichtenstein [1] or for a more modern discussion Lindblom [2]). The proof for general-relativistic stellar models has been more illusive and is still incomplete. Significant progress has been made recently, however, toward a proof in the relativistic case by Masood-ul-Alam [3, 4], Lindblom [5], Beig and Simon [6, 7], and Lindblom and Masood-ul-Alam [8]. These discussions show (under various assumptions) that the spatial geometry of static stellar models must be conformally flat as a consequence of the positive mass theorem. Since spatial conformal flatness is equivalent to spherical symmetry in static stellar models (Lindblom [9]), these arguments would be complete proofs of the spherical symmetry conjecture if they did not rely on unphysical assumptions. The purpose of this paper is to remove one of these "technical" assumptions and to weaken the unphysical restrictions on the equation of state of the fluid. We present the first complete proof of the necessity of spherical symmetry for static relativistic stellar models that are composed of a fluid whose adiabatic index satisfies certain inequalities.