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Number Variance for Arithmetic Hyperbolic Surfaces

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Abstract: We prove that the number variance for the spectrum of an arithmetic surface is highly nonrigid in part of the universal range. In fact it is close to having a Poisson behavior. This fact was discovered numerically by Schmit, Bogomolny, Georgeot and Giannoni. It has its origin in the high degeneracy of the length spectrum, first observed by Selberg.

1. Introduction

Let $\lambda_0 \leq \lambda_1 \leq \lambda_2 \dots$ be a sequence of numbers satisfying

$$N(x) = |\{j \mid \lambda_j \leq x\}| \sim x \quad \text{as} \quad x \to \infty . \tag{1.1}$$

There are many statistics that may be used to measure the fine structure of the distribution of the λ's. The one that we will use here is the number variance $\sum^2(\lambda, L)$ defined by

$$\sum{}^2(\lambda, L) = \langle (N(\lambda + L) - N(\lambda) - L)^2 \rangle , \tag{1.2}$$

where $\langle\,\rangle$ denotes local averaging in λ. $\sum^2(L)$ measures the variance from the expected number of "levels λ" lying in intervals of length L. For the local average we choose

$$\sum{}^2(\lambda, L) = \frac{1}{\lambda} \int\limits_{\lambda}^{2\lambda} (N(\xi + L) - N(\xi) - L)^2 \, d\xi . \tag{1.3}$$

Of course, we could replace 2 by c, where $c > 1$.

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