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Local Borel Summability of Euclidean Φ_4^4: A Simple Proof via Differential Flow Equations

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Abstract: It is shown how the differential flow equation (or, equivalently, the continuous renormalization group) method can be employed to give an astonishingly easy proof of the local Borel summability of the renormalized perturbative Euclidean massive Φ_4^4.

1. Introduction

Understanding rigorously (sometimes even only part of) the large order behaviour of perturbation theory in quantum field theory has proved to be quite a challenge (see e.g. chapter II.6 in [R] for a general review and references). For instance, let us consider the perturbative Euclidean massive Φ_d^4.

For $d \in \{2, 3\}$ it is known since a few years that the renormalized perturbation series for the connected Green functions is Borel summable and that the Borel transform exhibits an instanton singularity precisely as predicted by the Lipatov argument; the proof is based on constructive field theory techniques.

However, when $d = 4$, even the most sophisticated methods do not seem to be sufficient to go beyond a proof of the local existence of the Borel transform. In more detail, the combinatorially involved machinery of either elaborate BPHZ techniques [dCR] or discrete renormalization group/GN tree expansion methods [GN] proved adequate to establish local Borel summability, but without a good estimate of the minimal radius of convergence of the Borel transform. It required the introduction of multi-scale phase-space cluster expansion methods [MNRS, DFR] to obtain largely improved estimates (in fact, the suspected best possible estimates) on the radius of convergence. All attempts to prove the existence (or, less expected, the absence) of instanton or renormalon singularities failed, so far.

The purpose of this paper is to demonstrate that, somewhat unexpectedly, there is an easy and rather short proof of the local existence of the Borel transform for $d = 4$

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