

Dilogarithm Identities in Conformal Field Theory and Group Homology

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To Professor C. N. Yang for his 70th birthday

Abstract: Recently, Rogers' dilogarithm identities have attracted much attention in the setting of conformal field theory as well as lattice model calculations. One of the connecting threads is an identity of Richmond–Szekeres that appeared in the computation of central charges in conformal field theory. We show that the Richmond–Szekeres identity and its extension by Kirillov–Reshetikhin (equivalent to an identity found earlier by Lewin) can be interpreted as a lift of a generator of the third integral homology of a finite cyclic subgroup sitting inside the projective special linear group of all 2×2 real matrices viewed as a *discrete* group. This connection allows us to clarify a few of the assertions and conjectures stated in the work of Nahm–Recknagel–Terhoven concerning the role of algebraic K-theory and Thurston's program on hyperbolic 3-manifolds. Specifically, it is not related to hyperbolic 3-manifolds as suggested but is more appropriately related to the group manifold of the universal covering group of the projective special linear group of all 2×2 real matrices viewed as a topological group. This also resolves the weaker version of the conjecture as formulated by Kirillov. We end with a summary of a number of open conjectures on the mathematical side.

0. Introduction

Very recently, much has been written about the Rogers' dilogarithm identities and its role in conformal field theory, see [BR, KKMM, FS, K, KR, KP, KN, KNS, NRT]. For an excellent general survey for mathematicians concerning hypergeometric functions algebraic K-theory, algebraic geometry and conformal field theory, see [V] and its extensive section of references. For a recent review from the physics side, see [DKKMM]. In the present work, we limit our attention to the special case of dilogarithm identities. In spirit, it fits into the program surveyed by

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