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The Classification of Affine $SU(3)$ Modular Invariant Partition Functions

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Abstract: A complete classification of the *physical* modular invariant partition functions for the WZNW models is known for very few affine algebras and levels, the most significant being all levels of $SU(2)$, and level 1 of all simple algebras. In this paper we solve the classification problem for $SU(3)$ modular invariant partition functions, all levels. Our approach will also be applicable to other affine Lie algebras, and we include some preliminary work in that direction, including a sketch of a new proof for $SU(2)$.

1. Introduction

The classification of all rational conformal field theories (RCFTs) is clearly a desirable pursuit. In spite of tremendous progress in our understanding of RCFTs, we still find ourselves far from our ultimate goal. The problem can be somewhat simplified by focusing on the building blocks, the Wess–Zumino–Novikov–Witten (WZNW) models [41, 28, 16] associated with simple Lie algebras. Unfortunately, a full classification of even these models is still lacking. Only in the special cases of $SU(2)_k$ [6, 23, 15] and level 1 for all simple affine algebras [19, 9, 11] has a list of *physical* modular invariant partition functions been proven to be complete. The generalization of these proofs to higher ranks and levels has been plagued with difficulties due to the explosively increasing numbers of *non-physical* modular invariants. In this article we attempt to develop the tools necessary for this generalization, and successfully apply the new technique to $SU(3)_k$.

The partition function of a WZNW conformal field theory associated with affine Lie algebra (= current Lie algebra on S^1) [20, 25, 3] $\hat{g}$ and level k can be written as

$$Z = \sum N_{\lambda_L \lambda_R} \chi^k_{\lambda_L} \chi^{k*}_{\lambda_R} \,. \tag{1.1}$$

χ^k_λ is the *normalized character* [21] of the representation of $\hat{g}$ with (horizontal) highest weight λ and level k; it is a function of a complex vector z and a complex number τ. The algebra $\hat{g}$ is the untwisted affine extension $g^{(1)}$ of a simple Lie