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String Equations for the Unitary Matrix Model and the Periodic Flag Manifold

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Abstract: The periodic flag manifold (in the Sato Grassmannian context) description of the modified Korteweg–de Vries hierarchy is used to analyse the translational and scaling self-similar solutions of this hierarchy. These solutions are characterized by the string equations appearing in the double scaling limit of the symmetric unitary matrix model with boundary terms. The moduli space is a double covering of the moduli space in the Sato Grassmannian for the corresponding self-similar solutions of the Korteweg–de Vries hierarchy, i.e. of stable 2D quantum gravity. The potential modified Korteweg–de Vries hierarchy, which can be described in terms of a line bundle over the periodic flag manifold, and its self-similar solutions corresponds to the symmetric unitary matrix model. Now, the moduli space is in one-to-one correspondence with a subset of codimension one of the moduli space in the Sato Grassmannian corresponding to self-similar solutions of the Korteweg–de Vries hierarchy.

1. Introduction

In the last few years matrix models have received much attention as a non-perturbative formulation of string theory. These models can be described in the double scaling limit in terms of solutions to certain integrable systems. For the Hermitian matrix model (HMM) it was found [3] that in the double scaling limit the specific heat of the theory is a solution to the Korteweg–de Vries (KdV) hierarchy. This solution must satisfy also the string equation which is a self-similarity condition under Galilean symmetry transformations. This result was achieved by the use of orthogonal polynomials on the real line. The string equation can be written in terms of two scalar differential operators P, Q as

$$[P, Q] = \mathrm{id}\,.$$

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