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$N = 2$ Supersymmetric Quantum Mechanics on Riemann Surfaces with Meromorphic Superpotentials

N. V. Borisov[1,⋆], **K. N. Ilinski**[2]

[1] Institute of Physics, Saint Petersburg University, Ulianovskaya 1, 198904 Saint Petersburg, Russian Federation
[2] Saint Petersburg Steklov Mathematical Institute, Fontanka 27, 191011 Saint Petersburg, Russian Federation

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Abstract. We construct $N = 2$ supersymmetric quantum Hamiltonians with meromorphic superpotentials on compact Riemann surfaces and investigate the topological properties of these Hamiltonians. L_2-cohomology groups for supercharge (a deformed $\bar{\partial}$ operator) are considered and the Witten index for the supersymmetric Hamiltonian with meromorphic superpotential is calculated in terms of Euler characteristic of the Riemann surface and the degree of a divisor of poles for the differential of the superpotential.

1. Introduction

The conception of supersymmetry was introduced as a theoretical construction in the quantum theory of fundamental interactions [1]. Now this conception has useful applications in quantum mechanics [2] and mathematical physics [3] as a basis for the investigations of topological properties of the Hamiltonians and elliptic complexes. Supersymmetric scattering theory [4] gives a general approach to generalizations of index theorems on elliptic operators with continuous spectrum and to the investigation of topological properties of scattering matrices.

It is especially important to investigate the supersymmetric Hamiltonian H with nontrivial Witten index $\Delta_W(H)$ [or supersymmetric scattering index $n(H, H_0)$]. The classical Hodge-de Rham theory states that the Witten index for the Laplace operator on forms on the compact Riemannian manifold with smooth metric is equal to the Euler characteristic of the manifold. Another example of the supersymmetric Hamiltonian with nontrivial Witten index arises in the holomorphic $N = 2$ Wess-Zumino model in supersymmetric quantum mechanics which was introduced in [5, 6] and developed in [7–9]. It was discovered in [5] for the supersymmetric Hamiltonian on a (noncompact) complex plane with polynomial superpotential f the Witten index

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