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Lattice Topological Field Theory in Two Dimensions

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Abstract. The lattice definition of a two-dimensional topological field theory (TFT) is given generically, and the exact solution is obtained explicitly. In particular, the set of all lattice topological field theories is shown to be in one-to-one correspondence with the set of all associative algebras R, and the physical Hilbert space is identified with the center $Z(R)$ of the associative algebra R. Perturbations of TFT's are also considered in this approach, showing that the form of topological perturbations is automatically determined, and that all TFT's are obtained from one TFT by such perturbations. Several examples are presented, including twisted $N = 2$ minimal topological matter and the case where R is a group ring.

1. Introduction

Any consistent quantum field theory is expected to be realized as a continuum limit of a lattice model. Furthermore, the lattice definition is the only known method to investigate the non-perturbative structure of quantum field theories.

In this paper, we show that 2D topological field theories (TFT's), especially topological matter systems, can also be realized as lattice models, which will be called *lattice topological field theories* (LTFT's). The advantage of this approach to TFT over the conventional continuum field theoretic one [1] is in that this lattice definition makes much easier the understanding of geometric and algebraic structure of TFT. Moreover, since there should not be any dimensionful parameters in TFT or in LTFT, we do *not* need to take a continuum limit in our lattice model. This fact allows easy calculation of various quantities.

We first recall the basic axiom of TFT. Let $\hat{g}_{\mu\nu}$ be a background metric on a surface, on which matter field X_{matter} lives. The partition function $Z[\hat{g}_{\mu\nu}]$ is defined by

$$Z[\hat{g}] \equiv \int \mathscr{D}_{\hat{g}} X_{\text{matter}} \exp(-S[X_{\text{matter}}, \hat{g}]) , \tag{1.1}$$

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