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A Geometrical Presentation of the Surface Mapping Class Group and Surgery

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Abstract. We construct a tangle presentation of the mapping class group similar to a natural presentation of the braid group by geometrical braids. A relation between surgery and Heegard diagrams for 3-manifolds arising in this way and different applications are studied.

1. Introduction

It is well-known that the mapping class group of the disc with n marked points has a natural presentation as the group of geometrical braids with n strings. We give a similar presentation of the mapping class group of an orientable surface of arbitrary genus (which may also easily be generalized for the case of a surface with marked points). A relation between surgery presentation of 3-manifolds and Heegard diagrams (see [6, 11]) arising in this way is investigated. This relation enables us to prove that if a 3-manifold has Heegard decomposition of genus two, it may be obtained by surgery on a framed arborescent link in S^3. We also provide a new proof (similar in spirit to [7]) of Kirby's theorem [5], which in our setting is an easy consequence of stable equivalence of Heegard splittings and Wajnryb's presentation for the mapping class group of a surface [13].

The paper is organized in the following way: in Sect. 1 we recall the notion of framed $2n$-tangles and their diagrams. In Sect. 2 Kirby calculus for framed $2n$-tangles is introduced. Section 3 is devoted to the definition of the group T_{2n} of admissible $2n$-tangles. We state our main theorem in Sect. 4; the proof is given in Sects. 5, 6. In Sects. 7, 8 we study the relation between surgery and Heegard decompositions. As a corollary of our construction in a particular case of Heegard genus two we obtain (in Sect. 7) the result mentioned above. A new proof of Kirby's theorem is established in Sect. 9.

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