The Conformal Factor in the SAS Einstein–Maxwell Field Equations and a Central Extension of a Formal Loop Group

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Abstract: We consider a relation between the conformal factor in the stationary axisymmetric (SAS) Einstein–Maxwell field equations and a central extension of a formal loop group which is described by a group 2-cocycle on the formal loop group. The corresponding 2-cocycle on the Lie algebra of the formal loop group is the one which describes an affine Lie algebra. As a result, we see that the space of formal solutions with conformal factors is a homogeneous space of a central extension of the Hauser group.

0. Introduction

In [HS] we have discussed a σ-model with values in $S(U(1) \times U(2)) \backslash SU(1, 2)$ which is derived from the stationary axisymmetric (SAS) Einstein–Maxwell field equations. We formulated the theory of the σ-model in the category of formal power series by using Takasaki's formal loop group technique [T] and the linearization procedure investigated by Breitenlohner and Maison [BM]. However, we did not incorporate the conformal factor into the theory, neither did we state the homogeneous structure of the space of solutions of the Einstein–Maxwell field equations in stationary axisymmetric space-time.

As to the conformal factor, the second author, in [S], reproduced the results of [BM] in the category of formal power series and obtained an infinite dimensional homogeneous space structure of the space of formal solutions in the case of the Einstein equations.

In the present paper, following [BM, HS, S], we extend the theory of our σ-model to the Einstein–Maxwell field equations with N abelian gauge fields in stationary axisymmetric space-times involving the conformal factor. We prove that there is an elegant relation between the conformal factor and a group 2-cocycle on the formal loop group with values in $SU(1, N + 1)$, and show that the trivial central

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