Commun Math Phys 159, 593–604 (1994)

Communications in
Mathematical
Physics

The Existence and Uniqueness of Solutions of Yang-Mills Equations with Bag Boundary Conditions

Jędrzej Śniatycki⋆**, Günter Schwarz**⋆⋆

Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta, Canada

Received: 5 January 1993/in revised form: 23 March 1993

Abstract: The Cauchy problem for the Yang-Mills equations in the Coulomb gauge is studied on a compact, connected and simply connected Riemannian manifold with boundary. An existence and uniqueness theorem for the evolution equations is proven for fields with Cauchy data in an appropriate Sobolev space. The proof is based the Hodge decomposition of the Yang-Mills fields and the theory of non-linear semigroups.

1. Introduction

Quantum theory is usually formulated in a way which depends on the global structure of space. On the other hand it is supposed to describe phenomena in the atomic and subatomic scale. Hence, it is of interest to study the quantum theory of systems of finite spatial extension, and the role played by the boundary conditions.

Yang-Mills theory is a non-linear generalization of electrodynamics. Yang-Mills fields are connections in a right principal fibre bundle $\mathscr{P}$ over the space time manifold $X = M \times \mathbb{R}$ with structure group G describing the internal symmetries of the theory. The canonical variables in the Yang-Mills theory can be described as a pair of $\mathfrak{g}$-valued, time dependent 1-forms $A = A_i dx^i$ and $E = E_i dx^i$ on a typical 3 dimensional Cauchy surface M, where $\mathfrak{g}$ is the Lie algebra of the structure group. We assume that $\mathfrak{g}$ is equipped with an ad-invariant metric.

The Yang-Mills equations split into the evolution equations and the constraint equations. The constraint equation is

$$\delta E + [A\cdot, E] = 0\,, \tag{1.1}$$

where δ denotes the co-differential with respect to a given Riemannian metric g on M, $[\cdot,\cdot]$ denotes the Lie bracket in $\mathfrak{g}$, and the dot denotes the scalar product of forms,

⋆ Work partially supported by NSERC Research Grant A 8091
⋆⋆ Work supported by DFG Grant Schw 485/2-1