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Cyclic Monodromy Matrices for $sl(n)$ Trigonometric R-Matrices

Vitaly Tarasov
Physics Department, Leningrad University, Leningrad 198904, Russia

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Abstract: The algebra of monodromy matrices for $sl(n)$ trigonometric R-matrix is studied. It is shown that a generic finite-dimensional polynomial irreducible representation of this algebra is equivalent to a tensor product of L-operators. Cocommutativity of representations is discussed and intertwiners for factorizable representations are written through the Boltzmann weights of the $sl(n)$ chiral Potts model.

Introduction

Let us consider an algebra generated by noncommutative entries of the matrix $T(u)$ satisfying the famous bilinear relation originated from the quantum inverse scattering method [13, 20]

$$R(\lambda - \mu)T(\lambda)T(\mu) = T(\mu)T(\lambda)R(\lambda - \mu),$$

where $R(\lambda)$ is R-matrix – a solution of the Yang–Baxter equation. For historical reasons this algebra is called the algebra of monodromy matrices. It possesses a natural bialgebra structure with the coproduct (1.5). If $\mathfrak{g}$ is a simple finite-dimensional Lie algebra and $R(\lambda)$ is a $\mathfrak{g}$-invariant R-matrix the algebra of monodromy matrices after a proper specialization gives the Yangian $Y(\mathfrak{g})$ introduced by Drinfeld [11]. If $R(\lambda)$ is the corresponding trigonometric R-matrix [2, 14] (see (1.1) for $sl(n)$ case) this algebra is closely connected with $U_q(\mathfrak{g})$ and $U_q(\hat{\mathfrak{g}})$ at zero level [11, 14, 15, 22, 23]. In the last case it is convenient to use a new variable $u = \exp\lambda$ rather than λ. If $R(\lambda)$ is $sl(2)$ elliptic R-matrix [1, 5] the algebra of monodromy matrices gives rise to Sklyanin's algebra [24].

In this paper we shall study algebras of monodromy matrices for $sl(n)$ trigonometric R-matrices [6, 19, 21]. In the framework of the quantum inverse scattering method finite-dimensional irreducible representations of these algebras which depend polynomially on the spectral parameter u are of special interest. They correspond to integrable models on a finite lattice. L-operators are irreducible representations with linear dependence on the spectral parameter, and usually we