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Nonpersistence of Breather Families for the Perturbed Sine Gordon Equation

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Abstract. We show that, up to one exception and as a consequence of first order perturbation theory only, it is impossible that a large portion of the well-known family of breather solutions to the sine Gordon equation could persist under any nontrivial perturbation of the form

$$u_{tt} - u_{xx} + \sin u = \varepsilon \Delta(u) + O(\varepsilon^2),$$

where Δ is an analytic function in an *arbitrarily small* neighbourhood of $u = 0$. Improving known results, we analyze and overcome the particular difficulties that arise when one allows the domain of analyticity of Δ to be small. The single exception is a one-dimensional linear space of perturbation functions under which the full family of breathers does persist up to first order in ε.

1. Introduction

1.1. The Problem

Nontrivial solutions to a wave equation are called *breathers* if they decay as $|x| \to \infty$ (x the space variable) and are periodic in time t. A wave equation known to admit such solutions is the sine Gordon equation

$$u_{tt} - u_{xx} + \sin u = 0. \tag{1.1}$$

As it can be viewed as a completely integrable Hamiltonian system, explicit solutions are known, in particular the family of breathers

$$u^*(x,t) = u^*(x,t;m) = 4 \arctan \frac{m}{\omega} \frac{\sin \omega t}{\cosh mx}, \qquad m, \omega > 0,\ m^2 + \omega^2 = 1. \tag{1.2}$$

We are concerned with the question whether this type of solution exists for other nonlinear Klein Gordon equations

$$u_{tt} - u_{xx} + g(u) = 0, \qquad (g(0) = 0, g'(0) \neq 0) \tag{1.3}$$