Commun. Math. Phys. 158, 373–396 (1993)

Communications in Mathematical Physics

Semiclassical Approximation in Batalin-Vilkovisky Formalism

Albert Schwarz

Department of Mathematics, University of California, Davis, CA 95616, USA
E-mail: ASSCHWARZ@UCDAVIS.EDU

Received: 29 October 1992

Abstract. The geometry of supermanifolds provided with a Q-structure (i.e. with an odd vector field Q satisfying $\{Q, Q\} = 0$), a P-structure (odd symplectic structure) and an S-structure (volume element) or with various combinations of these structures is studied. The results are applied to the analysis of the Batalin-Vilkovisky approach to the quantization of gauge theories. In particular the semiclassical approximation in this approach is expressed in terms of Reidemeister torsion.

0. Introduction

The Batalin-Vilkovisky formalism (BV-formalism) is based on a notion of an odd Poisson bracket (antibracket). The odd Poisson bracket of two functions F, G on the (super)space $R^{n,n}$ with even coordinates $x^1, \dots, x^n$ and odd coordinates $\xi_1, \dots, \xi_n$ can be defined by the formula

$$\{F, G\} = \frac{\partial F}{\partial x^a}\frac{\partial_l G}{\partial \xi_a} - \frac{\partial_r F}{\partial \xi_a}\frac{\partial G}{\partial x^a}\,. \tag{1}$$

The transformations of $R^{n,n}$ preserving the bracket (1) are called odd symplectic transformations or P-transformations. Volume preserving P-transformations are called SP-transformations. A manifold X pasted together from domains in $R^{n,n}$ by means of P-transformations is called an odd symplectic manifold or a P-manifold. Replacing P-transformations by SP-transformations in this definition we get a notion of an SP-manifold. In a P-manifold X we have a notion of an odd Poisson bracket $\{F, G\}$ of two functions on X; in an arbitrary coordinate system we can express $\{F, G\}$ as

$$\{F, G\} = \frac{\partial_r F}{\partial z^i}\omega^{ij}(z)\frac{\partial_l G}{\partial z^j}\,.$$

The 2-form $\omega = dz^i \omega_{ij}(z) dz^j$ is closed. (Here the matrix ω_{ij} is inverse to ω^{ij}.) The formula

$$\omega(\zeta, \widetilde{\zeta}) = \zeta^i \omega_{ij}(z) \widetilde{\zeta}^j$$