

Ferromagnetism in the Hubbard Model

Examples from Models with Degenerate Single-Electron Ground States

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Abstract. Whether spin-independent Coulomb interaction can be the origin of a realistic ferromagnetism in an itinerant electron system has been an open problem for a long time. Here we study a class of Hubbard models on decorated lattices, which have a special property that the corresponding single-electron Schrödinger equation has N_d -fold degenerate ground states. The degeneracy N_d is proportional to the total number of sites $|\Lambda|$. We prove that the ground states of the models exhibit ferromagnetism when the electron filling factor is not more than and sufficiently close to $\varrho_0 = N_d/(2|\Lambda|)$, and paramagnetism when the filling factor is sufficiently small. An important feature of the present work is that it provides examples of three dimensional itinerant electron systems which are proved to exhibit ferromagnetism in a finite range of the electron filling factor.

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