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Internal Lifschitz Singularities for One Dimensional Schrödinger Operators

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Abstract. The integrated density of states of the periodic plus random one-dimensional Schrödinger operator $H_\omega = -\Delta + V_{\text{per}} + \sum_i q_i(\omega) f(\circ - i)$; $f \geq 0$, $q_i(\omega) \geq 0$, has Lifschitz singularities at the edges of the gaps in $Sp(H_\omega)$. We use Dirichlet-Neumann bracketing based on a specifically one-dimensional construction of bracketing operators without eigenvalues in a given gap of the periodic ones.

1. Introduction

In this paper we will consider the behavior of the integrated density of states (IDS) for the one-dimensional random Schrödinger operator.

$$\begin{aligned} H_\omega(g) &= -\Delta + V_{\text{per}} + gV_\omega \\ &= T + gV_\omega , \end{aligned} \tag{1.1}$$

where

$$V_{\text{per}}(x+1) = V_{\text{per}}(x) \tag{1.2}$$

is a periodic, piecewise continuous function, $g > 0$,

$$V_\omega(x) = \sum_{n\in\mathbb{Z}} q_n(\omega) f(x-n), \tag{1.3}$$

with piecewise continuous $f \geq 0$, $\operatorname{supp} f \subset \left(-\frac{1}{2}, \frac{1}{2}\right)$, and $q_n(\omega)$ are independent, identically distributed (iid) random variables. Their distribution function μ is assumed to have compact support

$$\operatorname{supp} \mu \subset [0, 1] \tag{1.4}$$

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