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Symmetry Breaking in Heisenberg Antiferromagnets

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Abstract: We extend Griffith's theorem on symmetry breaking in quantum spin systems to the situation where the order operator and the Hamiltonian do not commute with each other. The theorem establishes that the existence of a long range order in a symmetric (non-pure) infinite-volume state implies the existence of a symmetry breaking in the state obtained by applying an infinitesimal symmetry-breaking field. The theorem is most meaningful when applied to a class of quantum antiferromagnets where the existence of a long range order has been proved by the Dyson-Lieb-Simon method. We also present a related theorem for the ground states. It is an improvement of the theorem by Kaplan, Horsch and von der Linden. Our lower bounds on the spontaneous staggered magnetization in terms of the long range order parameter take into account the symmetry of the system properly, and are likely to be saturated in general models.

1. Introduction

Among the most important issues in rigorous statistical mechanics is to establish the existence of phase transitions in various idealized models of physical systems. When a transition is accompanied by a breakdown of a discrete symmetry, the Peierls' argument and its variants [1] can be applied to produce strong rigorous results. When a transition is accompanied by a breakdown of a continuous symmetry, on the other hand, the Peierl's method does not work in general. Fröhlich, Simon and Spencer [2] developed a method based on reflection positivity [3], which enabled them to prove the existence of phase transitions in various classical spin systems with continuous symmetry. The method was extended to quantum spin systems by Dyson, Lieb and Simon [4]. Their result is outstanding in that it rigorously establishes the existence of phase transitions in physically realistic models, such as the three-dimensional quantum Heisenberg antiferromagnets.

In order to motivate the theorem of the present paper, we shall describe the main result of Dyson, Lieb and Simon in the context of Heisenberg antiferromagnets. Take