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Modular Structure and Duality in Conformal Quantum Field Theory

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Abstract. Making use of a recent result of Borchers, an algebraic version of the Bisognano-Wichmann theorem is given for conformal quantum field theories, i.e the Tomita-Takesaki modular group associated with the von Neumann algebra of a wedge region and the vacuum vector coincides with the evolution given by the rescaled pure Lorentz transformations preserving the wedge. A similar geometric description is valid for the algebras associated with double cones. Moreover essential duality holds on the Minkowski space M, and Haag duality for double cones holds provided the net of local algebras is extended to a pre-cosheaf on the superworld $\tilde{M}$, i.e. the universal covering of the Dirac-Weyl compactification of M. As a consequence a PCT symmetry exists for any algebraic conformal field theory in even space-time dimension. Analogous results hold for a Poincaré covariant theory provided the modular groups corresponding to wedge algebras have the expected geometrical meaning and the split property is satisfied. In particular the Poincaré representation is unique in this case.

Introduction

Haag duality in Quantum Field Theory is the property that local observable algebras maximally obey the causality principle: if $\mathscr{R}(\mathscr{O})$ is the von Neumann algebra of the observables localized in the double cone $\mathscr{O}$ of the Minkowski space M, then $\mathscr{R}(\mathscr{O})$ is the commutant of the von Neumann algebra $\mathscr{R}(\mathscr{O}')$ of the observables localized in the space-like complement $\mathscr{O}'$ of $\mathscr{O}$,

$$\mathscr{R}(\mathscr{O}') = \mathscr{R}(\mathscr{O})' .$$

Duality plays an important role in the structural analysis of algebraic Quantum Field Theory [9] and has long been verified in free field models [1]. If the local

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