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The Full Diagonal Model of a Bose Gas

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Abstract. This paper is the final one in a series in which we investigate some models of an interacting Bose gas using Varadhan's large deviation version of Laplacian asymptotics; in it we study the equilibrium thermodynamics of the full diagonal model of a Bose gas. We obtain a formula expressing the pressure, in the thermodynamic limit, as the supremum of a functional over the space of positive bounded measures. We analyse this formula for a large class of interaction kernels and show that there is a critical temperature below which there is Bose–Einstein condensation.

1. Introduction

The Hamiltonian for a system of bosons interacting through a pair potential $\phi(x-x')$ can be written as

$$H = T + U\,, \tag{1.1}$$

where T is the kinetic energy operator and U is the potential energy operator,

$$U = \frac{1}{2}\int\int \phi(x-x')\psi^*(x)\psi^*(x')\psi(x)\psi(x')dxdx'\,, \tag{1.2}$$

where $\psi(x)$ and $\psi^*(x)$ satisfy the canonical commutation relations. For particles in a cube Λ of volume V in $\mathbb{R}^d$ with periodic boundary conditions, the Hamiltonian can be written in terms of momentum space operators using

$$\psi(x) = \frac{1}{V}\sum_k a_k e^{ikx} \quad \text{and} \quad v(k) = \int_\Lambda \phi(x)e^{-ikx}dx:$$

$$T = \sum_k \varepsilon(k)n_k \tag{1.3}$$

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